AN ANALYTICAL APPROACH TO THE EFFECTS OF VISCOUS DISSIPATION ON FORCED CONVECTIVE HEAT TRANSFER FOR A COUETTE-POISEUILLE FLOW OF POWER-LAW FLUID BETWEEN ASYMMETRIC BOUNDARY CONDITIONS

VISCOUS DISSIPATION

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ABSTRACT

An analytical approach to the effects of viscous dissipation on forced convective heat transfer for a Couette-Poiseuille flow of power-law fluid between asymmetric boundary conditions is considered. The studied Couette-Poiseuille flow is simply a maximum velocity flow and a numerical solution is required to find the region where the maximum velocity occurs. A new analytical formula for the Nusselt number was obtained regarding the heat flow ratio between upper and lower parallel plates and the Brinkman number established for a power law fluid. The results which condensed to a few specific examples are consistent with previous findings. If we compare the Nusselt number and the Brinkman number, we see an asymptotic Brinkman number. This has a sign shift for the set of power law indices under consideration.

Keywords: Viscous dissipation, Power-law fluids, Nusselt number, Constant heat flux.

Nomenclature

c_p	specific heat at constant pressure, J . kg^{-1} . K^{-1}
k_{eff}	effective thermal conductivities, W. m^{-1} . K^{-1}
L	channel width, m
$L_{\max}$	position of maximum velocity, m
$L^*_{\max}$	dimensionless maximum velocity coordinate
Nu	Nusselt number
n	power-law fluid index
P	pressure, Pa
Q_1, Q_2	wall heat flux, W . m^{-2}
T	fluid temperature, K
T_m	bulk mean fluid temperature, K
u	fluid velocity, m . s^{-1}
u_a	velocity profile above position of maximum velocity, m . s^{-1}
u^*_a	dimensionless fluid velocity at u_a
u_b	velocity profile below position of maximum velocity, m . s^{-1}

u_b^*	dimensionless fluid velocity at u_b
u_m	average velocity over channel width, m . s^{-1}
u_{up}	moving plate velocity,
u_{up}^*	dimensionless fluid velocity, u_{up}/u_m
u^*	dimensionless fluid velocity, u/u_m
x, y	Cartesian coordinates, m
y^*	dimensionless transverse coordinate
η	consistency factor, Pa . s^n
θ	dimensionless temperature
θ_A, θ_B	dimensionless temperature profile above (below) position with maximum velocity
θ_L	dimensionless bulk mean temperature at the top plate
θ_m	dimensionless bulk mean temperature
μ	fluid dynamic viscosity, Ns . m^{-2}
μ_{eff}	effective viscosity, Ns . m^{-2}
ρ	fluid density, kg . m^{-3}
τ	shear stress, N. m^{-2}

Introduction

In Couette-Poiseuille flow, where a moving surface continuously exchanges heat with the surrounding liquid, processes such as extrusion, metal forming, glass fiber drawing, and continuous casting play an important role [1]. The rheological behavior of fluids is important and can influence the quality of the materials being processed, but viscous dissipation (work done by viscous forces acting on the fluid) is also important in processes where fast gradients cause temperature increases. It could be important. .

The influence of viscous dissipation on forced convective heat transfer has been widely reported in the literature [1-7]. Aydin and Avci [1] studied the effect of dispersive dissipation on well developed convection heat transfer through a tube with constant temperature flux and continuously changing wall temperatures. When the Brinkman number Br is large, the temperature profile and the Nusselt number Nu are each greatly affected. An exact solution to the Graetz problem was obtained in the work by Ou and Cheng [2,3] on the thermal inlet region for viscous dissipation effects as forced convection. Aydin [4] solved the same problem, but used a different solution and assumed that axial conduction was negligible. Aydin and Avci [6] studied the thermal convection in the Poiseuille flow of Newtonian fluids and obtained an exact solution. In a study investigating the influence of boundary considerations, Aydin and Avci [1] analytically solved the temperature profile of a Couette-Poiseuille flow and showed that viscous dissipation effects are important. By taking into account asymmetric thermal boundary conditions, creating temperature solutions, and formulating Nusselt equations, Sheela-Francisca and Tso [7] continued their work. Results for Newtonian flows with stationary or moving boundaries are still limited.

There is related work that focuses on non-Newtonian flows. Payvar [8] studied the effects of viscous dissipation on power law liquids, Bingham plastic liquids, and Ellis liquids for thermally fully developed forced convection with constant wall heat flux. Power law and non-Newtonian fluids in Couette-Poiseuille flows between parallel plates were explored by Davaa et al. [9,10]. For flows that were exposed to constant heat fluxes applied to fixed and moving boundaries, respectively, accurate solutions for velocity and temperature were found in [9]. In another study [10], the modified power law model [11] defined by Irvine and Karni was used in the momentum and energy governing equations to improve the accuracy of the velocity field in the low shear rate regime . The underlying equations were solved numerically for a fully developed flow with constant heat flux. Hashemabadi et al. [12] used a simplified Phan Thien-Tanner model to take into account the viscous dissipation effect in Couette-Poiseuille flow between parallel plates for viscoelastic flow. Tso et al. [13] extended the work of [7] by considering the behavior of a power-law fluid in the analysis of forced convective heat transfer between solid parallel plates undergoing asymmetric heating at the top and bottom plates. Uwaezuoke and Ihekuna [14] derived a semi-analytical solution to the temperature distribution of his Couette-Poiseuille flow of pseudoplastic fluids. The temperature distribution and Nusselt number obtained with asymmetric heat flow boundary conditions are significantly influenced by the moving plate velocity, the power law exponent, the modified Brinkman number, and the heat flow ratio applied at the boundary along with dimensionless parameters representing constants, affects the integral when solving the momentum equation. However, the solution is complicated by the need to define this constant of integration.

Therefore, this study aims to improve the solution method in [14] and provide an analytical solution for heat transfer in Couette-Poiseuille flow subject to asymmetric heat flow boundary conditions, which is not often reported in the literature. Although the temperature and velocity distribution equations are accurate, the location corresponding to the maximum velocity must be solved numerically. This work will also extend the work of [14] by including a dilatant fluid solution. The velocity profile of Davaa et al. [9], and the solution requires the existence of a current maximum velocity, so the solution is limited to Couette-Poiseuille flows with maximum velocity.

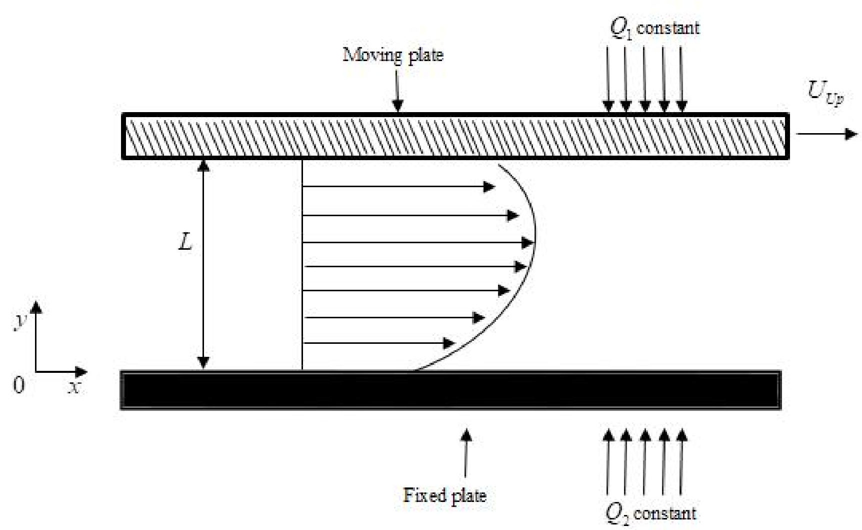

Fig. 1 Schematic diagram of the problem

Analysis and Problem Description

Consider a steady laminar non-Newtonian power-law fluid flowing through infinitely long parallel plates distanced apart from each other, as displayed in Fig. 1. The fluid is assumed to be incompressible and fully developed, both hydrodynamically and thermally, as well as having constant properties throughout. The upper plate is moving while both upper and lower plates are subjected to heat fluxes Q_1 and Q_2 respectively.

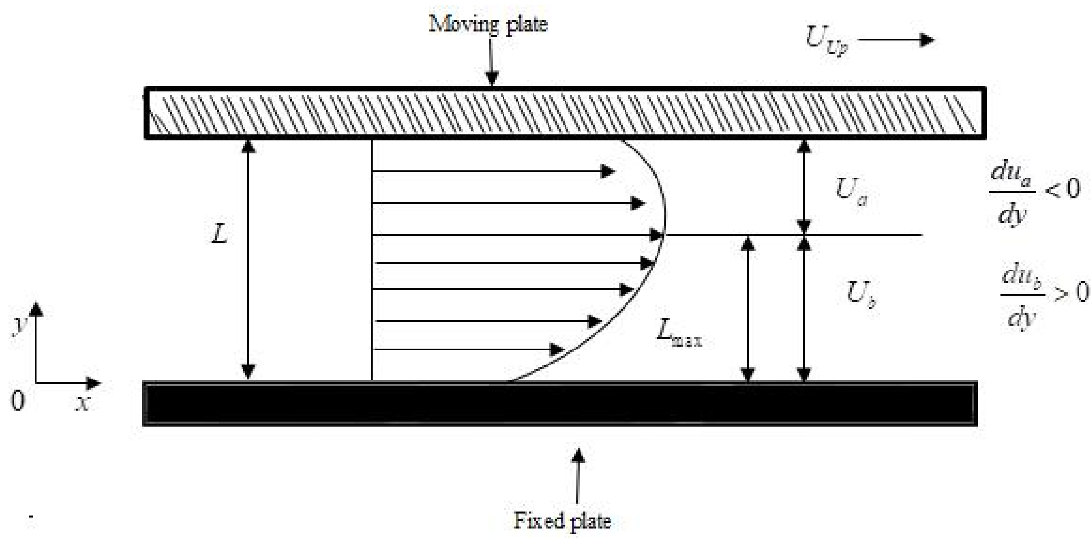

Fig. 2 Couette-Poiseuille flow problem

For a steady, incompressible, unidirectional and fully developed flow, Davaa et al [9] solved appropriately the non-dimensionalized momentum equation for a Cosette-Poiseuille flow. Flows with a maximum velocity between parallel plates are being considered where the vertical distance from the fixed plate to the location with the maximum velocity is denoted by $L_{\max}$ as displayed in Fig. 2. By dividing the flow into two parts denoted as u_a and u_b in Fig. 2. Davaa et al [9] obtained the analytical solution for the velocity profile in the parallel plate as

$$0 \le y^* \le L^*_{\max}$$

$$u^*_b = F^{1/n}\left(\frac{n}{n+1}\right)\left[L^{*\;\;n+1/n}_{\max} - \left(L^*_{\max} - y^*\right)^{n+1/n}\right] \tag{1}$$

$$L^*_{max} \le y^* \le 1$$

$$u^*_a = u^*_{up} + F^{1/n}\left(\frac{n}{n+1}\right)\left[\left(1 - L^*_{\max}\right)^{n+1/n} - \left(y^* - L^*_{\max}\right)^{n+1/n}\right] \tag{2}$$

where

$$y^* = \frac{y}{L}$$
$$u^* = \frac{u}{u_m}$$
$$u^*_{up} = \frac{u_{up}}{u_m}$$
$$L^*_{\max} = \frac{L_{\max}}{L}$$

and u_m is the mean velocity. By setting

$$u^*_b = u^*_a \text{ at } y^* = L^*_{\max} \tag{3}$$

F can be written in terms of $L^*_{\max}$ as

$$F = \left\{ \frac{u_{up}^*}{\left(n/_{2n} + 1 \right) \left[L_{\max}^{*\ n+1/_n} - \left(1 - L_{\max}^*\right)^{n+1/_n} \right]} \right\}^n \tag{4}$$

Next, by incorporating the continuity equation

$$\int_0^1 u^* dy^* = \int_0^{L_{\max}^*} u_b^* dy^* + \int_{L_{\max}^*}^1 u_a^* dy^* = 1 \tag{5}$$

the second relationship between F and $L_{\max}^*$ is obtained as

$$F = \left\{ \frac{1 - u_{up}^* \left(1 - L_{\max}^*\right)}{\left(n/_{2n} + 1 \right) \left[L_{\max}^{*\ 2n+1/_n} - \left(1 - L_{\max}^*\right)^{2n+1/_n} \right]} \right\}^n \tag{6}$$

By equating Eqs. (4) and (6), a relationship among $u_{up}^*, L_{\max}^*$ and n is developed:

$$u_{up}^* = \frac{L_{\max}^{*\ n+1/_n} - \left(1 - L_{\max}^*\right)^{n+1/_n}}{L_{\max}^{*\ n+1/_n} - \dfrac{n}{2n+1} \left[L_{\max}^{*\ 2n+1/_n} + \left(1 - L_{\max}^*\right)^{2n+1/_n} \right]} \tag{7}$$

Hence, $L_{\max}^*$ can be solved numerically if u_{up}^* and n are specified. To proceed with the beat transfer analysis, the energy equation is written as [2]

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} - \tau \left(\frac{du}{dy} \right) \tag{8}$$

For a unidirectional flow along the x direction, which has the velocity distribution as a function of y only, the shear stress of a power-law fluid may be described as

$$0 \le y \le L_{\max} \qquad \tau = -\eta \left(\frac{du}{dy} \right)^n \tag{9}$$

$$L_{\max} \le y \le L \qquad \tau = \eta \left(-\frac{du}{dy} \right)^n \tag{10}$$

The thermal boundaries in this study are

$$\left. \begin{aligned} -k \frac{\partial T}{\partial y} = Q_2, \quad \text{at } y = 0 \\ k \frac{\partial T}{\partial y} = Q_1, \quad \text{at } y = L \end{aligned} \right\} \tag{11}$$

For a fully developed flow, integrating Eq. (8) over the entire width of the channel subjected to the thermal boundaries in Eq. (11) yielded the axial temperature gradient, which can be expressed as

$$\frac{\partial T}{\partial x} = \frac{Q_1}{\rho c_p u_m L} \left[\left(1 + \frac{Q_2}{Q_1} \right) - \frac{\int_0^L \left(\frac{du}{dy} \right) \tau dy}{Q_1} \right] \tag{12}$$

Introducing another two dimensionless parameters

$$\theta = \frac{T}{Q_1 L / k} \tag{13}$$

$$Br = \frac{\eta u_m^{n+1}}{Q_1 L^n} \tag{14}$$

and by incorporating the respective Eqs. (9) and (10) into Eq. (8), the following dimensionless energy equation is obtained:

$$\frac{d^2\theta}{dy^{*2}} = u^*\left(1+\frac{Q_2}{Q_1}\right) + Br\left[u^*\left(\int_0^1 \left|\frac{du^*}{dy^*}\right|^{n+1} dy^*\right) - \left|\frac{du^*}{dy^*}\right|^{n+1}\right] \tag{15}$$

Denoting the temperatures above and below the position with maximum velocity as θ_A and θ_B, the respective boundary conditions for each temperate are

$$\frac{d\theta_b}{dy^*} = -\frac{Q_2}{Q_1} \text{ at } y^* = 0 \tag{16a}$$

$$\theta_b = \theta_a \text{ at } y^* = L^*_{\max} \tag{16b}$$

$$\left.\frac{d\theta_A}{dy^*}\right|_{y^*=L^*_{\max}} = \left.\frac{d\theta_B}{dy^*}\right|_{y^*=L^*_{\max}} \quad \text{at } y^* = L^*_{\max} \tag{16c}$$

$$\theta_a = \theta_L \text{ at } y^* = 1 \tag{16d}$$

A. Analytical Solution for the Temperature Profile

Solving Eq. (15) subject to Eq (16a-16d) with the given velocity profiles in Eqs. (1) and (2), we obtain solution as

$$0 \le y^* \le L_{\max} \quad \theta_A = \theta_a + \theta_{aBr} \tag{17}$$

$$\theta_a = \theta_L + C_1 y^{*2} + C_2 y^* + C_3 - C_4\left(y^* - L^*_{\max}\right)^{3n+1/n} \tag{18}$$

$$\theta_{aBr} = Br\left[C_1' y^{*2} + C_2' y^* + C_3' - \left(C_4' - C_5'\right)\left(y^* - L^*_{\max}\right)^{3n+1/n}\left(L^*_{\max} - y^*\right)^2\right] \tag{19}$$

For $L^*_{\max} \le y^* \le 1$

$$\theta_B = \theta_b + \theta_{bBr} \tag{20}$$

$$\theta_b = \theta_L + C_5 y^{*2} + C_6 y^* + C_7 - C_8 \left(L^*_{\max} - y^*\right)^{3n+1/n} \tag{21}$$

$$\theta_{bBr} = Br\left[C'_6 y^{*2} + C'_7 y^* + C'_8 - C'_9 \left(L^*_{\max} - y^*\right)^{3n+1/n}\right] \tag{22}$$

where the coefficients $C_1 - C_8$ and $C'_1 - C_9$ are a function of $n, L^*_{\max}, u^*_{up}, Br$ and Q_2/Q_1. The dimensionless bulk mean temperature is defined as

$$\theta_m = \frac{T_m}{Q_1 L/k} \tag{23}$$

B. Nusselt Number Expression

Defining Nusselt number Nu as

$$Nu = \frac{2LQ_1}{k\left(T_L - T_m\right)} = \frac{2}{\theta_L - \theta_m} \tag{24}$$

and evaluating Nu based on

$$\theta_L - \theta_m = \int_0^1 u^*\left(\theta_L - \theta\right) dy^* \text{ (25)}$$

gives the analytical expression for Nu. Because the defined Nu is bed in Q_1, it displays the top plate's convection rate. The Nusselt number for three different power indices is solved in the scenario of unequal heat flows. To address the Newtonian fluid, shear thinning, and shear thickening, we will be using $n=0.5, n=1.5$ and $n=1$, respectively, to represent each class of fluid. For each class, three respective velocities for the moving plate are solved, namely, for moving the plate in the favorable direction $\left(u_{up}^{*}=1\right)$, for the stationary plate $\left(u_{up}^{*}=0\right)$, and for the plate in the undesirable direction $\left(u_{up}^{*}=-1\right)$. The expressions for Nu are given nest for a specific u_{up}^{*}.

1. Shear thinning aids, $n=0.5$.

❖ Moving plate at $u_{up}^{*}=1$:

$$Nu=\left[0.144829-0.128846Br-0.0731288\left(\frac{Q_2}{Q_1}\right)\right]^{-1} \tag{25}$$

❖ Plate at $u_{up}^{*}=0$:

$$Nu=\left[0.182099+0.192054Br-0.0679012\left(\frac{Q_2}{Q_1}\right)\right]^{-1} \tag{26}$$

❖ Moving plate at $u_{up}^{*}=-1$:

$$Nu=\left[0.224744+0.997631Br-0.0565795\left(\frac{Q_2}{Q_1}\right)\right]^{-1} \tag{27}$$

2. Newtonian fluids, $n=1$.

❖ Moving plate at $u_{up}^{*}=1$:

$$Nu=\left[0.138095-0.190476Br-0.0702381\left(\frac{Q_2}{Q_1}\right)\right]^{-1} \tag{28}$$

❖ Plate at $u_{up}^{*}=0$:

$$Nu = \left[0.185714 + 0.385714 Br - 0.0642857 \left(\frac{Q_2}{Q_1} \right) \right]^{-1} \quad (29)$$

- ❖ Moving plate at $u_{up}^* = -1$:

$$Nu = \left[0.242857 + 2.74286 Br - 0.0488095 \left(\frac{Q_2}{Q_1} \right) \right]^{-1} \quad (30)$$

3. Shear thickening fluids, $n = 1.5$.

- ❖ Moving plate at $u_{up}^* = 1$:

$$Nu = \left[0.134889 - 0.288144 Br - 0.0688856 \left(\frac{Q_2}{Q_1} \right) \right]^{-1} \quad (31)$$

- ❖ Plate at $u_{up}^* = 0$:

$$Nu = \left[0.187343 + 0.7717 Br - 0.0626566 \left(\frac{Q_2}{Q_1} \right) \right]^{-1} \quad (32)$$

- ❖ Moving plate at $u_{up}^* = -1$:

$$Nu = \left[0.251755 + 7.1430 Br - 0.0450401 \left(\frac{Q_2}{Q_1} \right) \right]^{-1} \quad (33)$$

The analytical expressions for Nu reduce to the expressions for equal heat fluxes, when $Q_2/Q_1 = 1$. Taking $Q_2/Q_1 = 0$, the expressions are applicable to the insulated bottom boundary.

C. Nusselt Number Verification

The Nu expression is compared with those available in the literature to ascertain its validity. The Nu expression for Newtonian fluids and $u_{up}^* = 0$, that is, Eq. (29), agrees with Tso et al. [13]

when $Q_2 / Q_1 = 1$ and $Br = 0$. The Nu values for insulated bottoms boundaries, obtained from Eqs. (28-30), agree remarkably well with the numerical values solved by Davaa et al. [10] when $Q_2 / Q_1 = 0$ and the Brinkman number recast in the form defined by Eq. (14). The Nu expression fun $n = 0.5$ and $n = 1.0$, respectively, Eqs. (26) and (29), are compared with the exact solution in Tan and Chen [15] for $Q_2 / Q_1 = 1$ and $u_{up}^* = 0$. They are in exact agreement. Table 1 summarizes the Nu comparison.

Table 1 Comparison of the results for Nu

n	Br^*	Q_2 / Q_1	u_{up}^*	Nu^*, **Tso et al. [13]**	Nu, **Davaa et al. [9]**	Nu, **Tan and Chen [15]**	Nu, **present study**
1	0	0	0	5.385	5.385	-	5.385
1	0	1	0	8.235	-	8.235	8.235
1	0.2	0	0	-	3.804	-	3.804
1	0.2	0	1	-	10	-	10
1	0.2	0	-1	-	1.264	-	1.264
0.5	0.5	1	0	-	-	4.757	4.757
1.0	0.5	1	0	-	-	3.182	3.182

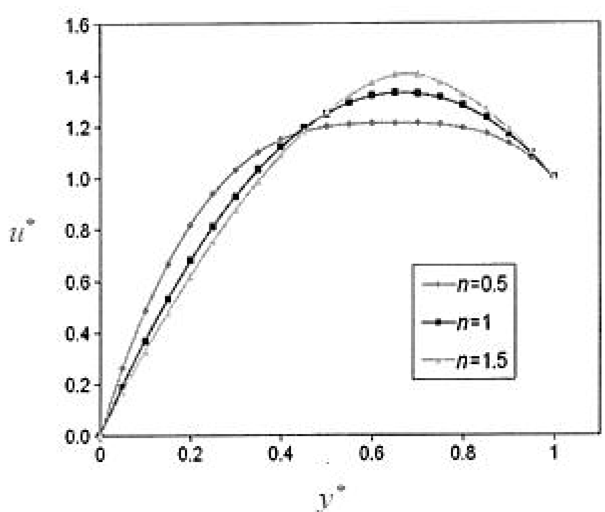

Fig. 3. Velocity profile along the transverse direction when $n = 0.5, 1$ and 1.5.

Discussion

Figure 3 displays the velocity profiles for a pseudoplastic $(n=0.5)$, a Newtonian fist $(n=1.0)$, and a shear thickening fluid $(n=1.5)$. Because the velocity profile would affect the heat convection rate, the characteristics of the velocity peddle is closely related to the temperature profile. The figure shows that, at the top moving wall, where $y^*=1$, the velocity gradient is highest for $n=1.5$ or shear thickening fluids, whereas at $y^*=0$, the velocity gradient is lowest for $n=1.5$. A higher velocity gradient at the wall is favorable to convection enhancement.

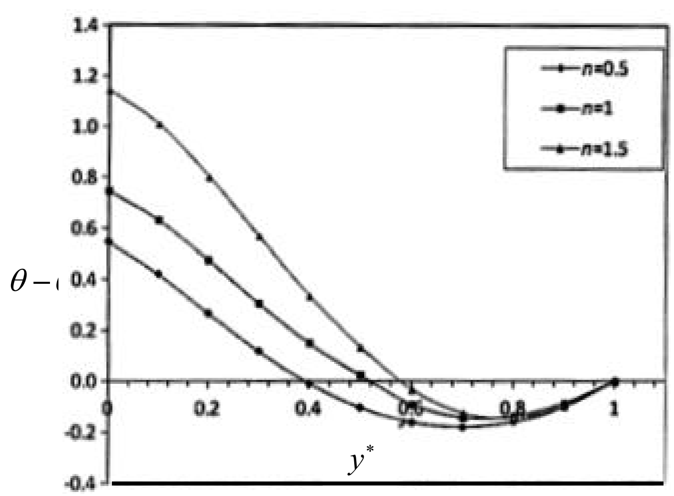

Fig. 4. Dimensionless temperature along the channel for $Br=0.5, u_{up}^*=1$ and $Q_2/Q_1=1.0$

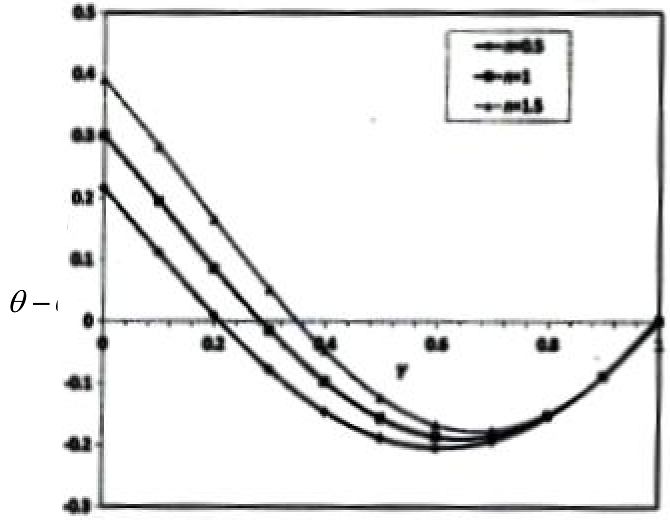

Fig. 5. Dimensionless temperature along the channel for $Br = 0.5, u_{up}^{*} = 1$ and $Q_2 / Q_1 = 1.0$

Figure 4 depicts the temperature profile for $n = 0.5, n = 1.0$ and $n = 1.5$ when $Br = 0.5, u_{up}^{*} = 1$, and $Q_2 / Q_1 = 1$. It is worth noting that, due to the asymmetric velocity, the heat convection at the top and bottom plates vary in different patterns as n increases. Understandably, from the characteristics of the velocity profile, the convection rate increases with decreasing n at the bottom fixed plate. Conversely, at the top moving plate, the heat convection rate increases, albeit not so significantly with increasing n. The temperature profile seen in Fig. 4 differs significantly from the one Tan and Chen [15] computed for a stable boundary, where the rate of heat convection increases at the border and decreases overall. Figure 5 displays the temperature profile for $Br = 0.1, u_{up}^{*} = 1$ and $Q_2 / Q_1 = 1$. A smaller Br as compared with Fig. 4 lowers the convection rate at the top moving plate, contrary to what is observed in a fixed boundary where the convection rate increases for a fixed boundary as Br is decreased. As a result, when a plate moves, the convection rate at the moving plate increases with more internal heat generation, or a higher Br as well as a larger n.

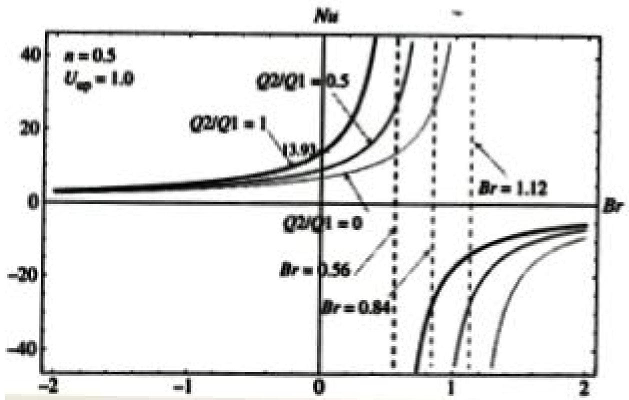

Fig. 6. Nusselt mumber versus Brinkmun number for $n = 0.5$.

Figure 6 displays Nu variation with Br for $n = 0.5$ a pieudoplastic fluid, for a moving top wall with $u_{up}^{*} = 1$. It is worth mentioning that Nu as defined in Eq. (24) is to evaluate the convection coefficient at the top moving plate. Hence, our focus should be on the effects of parameters such as Br and n on the heat convection rate at the top moving plate. The trend is alike for different ratios of heat fluxes, ${}^{Q_2}\!/_{Q_1}$, where Nu goes up with increasing ${}^{Q_2}\!/_{Q_1}$. On account of the first term on the right-hand side of Eq. (15), it is understandable that a larger heat flux ratio and velocity contribute to a larger heat convection rate. The figure reflects the variation of Nu with Br at a fixed ${}^{Q_2}\!/_{Q_1}$ presented by Eq. (25) where Nu increases with Br. It is important to note that the patters of Nu variation is different from the variation for a fixed boundary, as shown in Eq. (26) where Nu decreases with increasing Br. A top moving wall enhances the convection rate, causing a smaller difference between the bulk mean and wall temperature, hence a higher Nu. The Nu variation with Br in Fig. 6 can be explained from the asymmetric temperature and velocity profiles in Figs. 3 and 4, where the temperature is strongly dependent on Br and n.

It is important to note that there is an asymptotic Br as Br increases, manifested in the change in sign of Nu . The asymptote represents the Br that gives equal bulk mean and top wall temperatures. The asymptote can be obtained by equating the denominator of Eq(25) to zero for each specified Q_2/Q_1 . Notably, an increase in Q_2/Q_1 reduces the magnitude of the asymptote for Br . As Br increases beyond the asymptotic value, Nu changes sign. A negative Na indicates a higher bulk mean temperature than the top wall temperature, due to the larger amount of thermal energy stored, but not high enough heat convection rate as Br increases. A larger temperature difference in the temperatures between the bulk mean value and the top wall leads to a less negative Nu as Br increases beyond the asymptote.

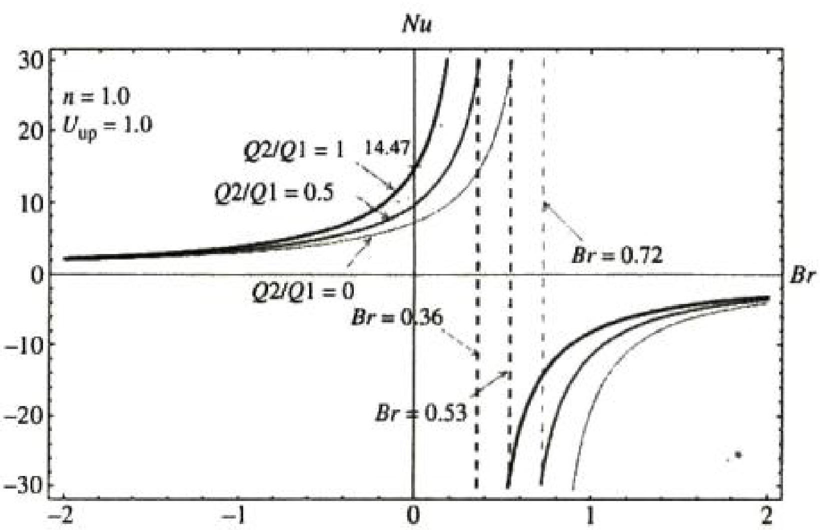

Fig. 7. Nusselt number versus Brinkman number for 16

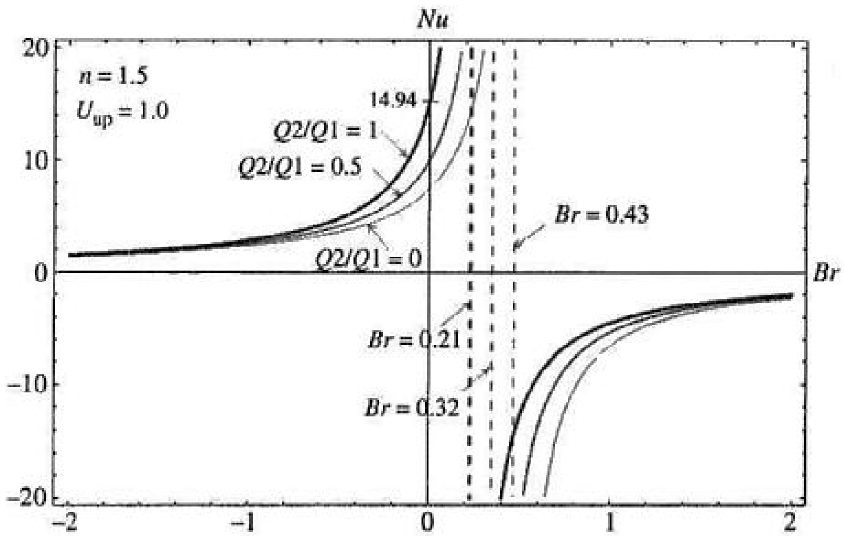

Fig. 8. Nusselt number versus Brinkman number for n = 1.5.

Figures 7 and 8 depict the respective Nu variation with Br for $n = 1$, a Newtonian fluid, and $n = 1.5$, a dilatant fluid. Although Figs. 7 and 8 share similar trends with Fig. 6, Figs. 7 and 8 show that, as n increases, Nu increases for a fixed Br and Q_2/Q_1, due to the higher heat convection rate. The values of the asymptotes decrease as n increases due to the growth in viscous dissipation. The top plate is the main focus of each of the figures described in this work, and they are qualitatively contrasted to the findings of Tan and Chen [15] for a fixed boundary, which demonstrate the opposite trend. The graphs' constant trend indicates that heat convection is enhanced by the axial movement of the upper plate in the direction of fluid velocity. The respective increase in Br and n has also improved the heat convection rate. However, once the asymptotic value of Br is exceeded, Nu changes to a negative sign owing to the large heat generation.

Conclusions

In this study, a power-law fluid heated asymmetrically at the boundaries of a laminar Couette-Poiseuille flow with fully developed thermal and hydrodynamic properties has been given an analytical temperature profile. By proposing a modified Brinkman number, attention has been drawn to the impact of viscous dissipation. The moving plate velocity and viscous dissipation, which in turn depend on the power-law index, have an impact on heat transfer in the parallel plates of a Couette-Poiseuille flow. The Nusselt number has a higher value when $u_{up}^{*} > 0$, with increasing Brinkman number, unlike $u_{up}^{*} = 0$, which shows an opposite trend. However, as Br increases beyond the asymptotic value, an increase in the viscous dissipation leads to a larger thermal energy stored and a negative Nu.

The top and bottom walls of a Couette flow display different patterns of temperature variation due to the moving boundary, as shown by the power-law index n is varied. Large velocity gradients toward the fixed wall, which are a characteristic of Couette flows, are caused by significant heat generation in the form of viscous dissipation. The velocity distribution toward the top moving wall has a considerable impact on the heat convection rate. Contrary to fixed boundaries, heat convection rate at the moving wall as indicated by Nu increases as Br and n increase.

By initially dividing the velocity profile at a place corresponding to the greatest velocity, the current study has successfully analytically solved the heat convection problem in a Couette flow. The Nu variation with Br shows a similar pattern for different ratios of heat fluxes imposed at the boundaries. However, the magnitude of Nu shows an increase with increasing ${Q_2}/{Q_1}$ at fixed.

The findings show that when the overall heat input to the flow increases, the moving boundary's rate of heat convection increases.

This study, in future may be extended to investigate the unavailable work due to entropy generation rate as n is varied and Br is increased, due to the enormous temperature gradients. Hence, in addition to solving the governing equations for mass, momentum, and energy, the entropy generation rate could be analyzed.

Appendix: Expressions for the Coefficients

$$C_1 = \left(\frac{Q_2}{Q_1}+1\right)\left[\frac{u_{up}^*}{2}+\frac{F^{1/n}(n)\left(1-L_{\max}^*\right)^{n+1/n}}{2(n+1)}\right] \tag{A1}$$

$$C_2 = \frac{F^{1/n}(n)\left(Q_2/Q_1+1\right)\left(L_{\max}^*\right)^{2n+1/n}}{2(n+1)} - \left(\frac{Q_2}{Q_1}+1\right)\left(\begin{array}{c} u_{up}^* L_{\max}^* + F^{1/n}(n)\left(L_{\max}^*\right) \\ \dfrac{\left(1-L_{\max}^*\right)^{n+1/n}}{(n+1)} \end{array}\right) - \left(\frac{Q_2}{Q_1}\right) \tag{A2}$$

$$C_3 = -C_2 - \frac{u_{up}^*}{2}\left(\frac{Q_2}{Q_1}+1\right)\frac{c1\left(F^{1/n}\right)(n)\left(L_{\max}^*-1\right)\left(Q_2/Q_1+1\right)\left(1-L_{\max}^*\right)^{1/n}}{2\left[(n+1)(2n+1)(3n+1)\right]} \tag{A3}$$

With $c1$ as

$$c1 = -2n^2 {L_{\max}^*}^2 + 4n^2 L_{\max}^* + 4n^2 + 5n + 1 \tag{A4}$$

$$C_4 = \frac{F^{1/n}(n)\left(y^*-L_{\max}^*\right)^{3n+1/n}\left(Q_2/Q_1+1\right)}{\left(2n+1/n\right)\left(3n+1/n\right)(n+1)} \tag{A5}$$

$$C_1' = \frac{A_1\left[u_{up}^* + u_{up}^* n + F^{1/n}(n)\left(1-L_{\max}^*\right)^{1/n}\right]}{2n+2} - \frac{A_1\left[F^{1/n}(n)\left(L_{\max}^*\right)\left(1-L_{\max}^*\right)^{1/n}\right]}{2n+2} \tag{A6}$$

$$C_2' = \frac{A_1\left[F^{1/n}(n)\left(L_{\max}^*\right)^{2n+1/n}\right]}{n+1} - \frac{A_1 n + F + F^{1/n}\left(n^{n+1}\right)(F)\left(L_{\max}^*\right)^{2n+1/n}}{(n+1)(2n+1)}$$
$$- \frac{A_1\left(L_{\max}^*\right)\left[u_{up}^* + u_{up}^* n + F^{1/n}(n)\left(1-L_{\max}^*\right)^{1/n} + F^{1/n}(n)\left(L_{\max}^*\right)\left(1-L_{\max}^*\right)^{1/n}\right]}{n+1} \tag{A7}$$

$$C_3' = \frac{\left(1-L_{\max}^*\right)^{3n+1/n}\left(n^2\right)\left[(-F)^{n+1/n} + n(-F)^{n+1/n} + A_1(n)F^{1/n}\right]}{(n+1)(2n+1)(3n+1)} \tag{A8}$$

$$C_4' = \frac{\left(-F^{n+1/n}\right)n^2}{5n+1+6n^2} \tag{A9}$$

$$C_5' = \frac{A_1\left(F^{1/n}\right)n^3}{(n+1)(2n+1)(3n+1)} \tag{A10}$$

where

$$A_1 = \frac{\left[\left(F^{n+1/n}\right)^{2n+1/n} + \left|-F^{n+1/n}\right|\left(1-L_{\max}^*\right)^{2n+1/n}\right]}{2n+1/n} \tag{A11}$$

$$C_5 = \left(\frac{Q_2}{Q_1}+1\right)\left[\frac{F^{1/n}(n)\left(L_{\max}^*\right)^{n+1/n}}{2(n+1)}\right] \tag{A12}$$

$$C_6 = -\frac{Q_2}{Q_1} - \left(\frac{Q_2}{Q_1}+1\right)\left[\frac{F^{1/n}\left(n^2\right)\left(L^*_{\max}\right)^{2n+1/n}}{\left(2n+1\right)\left(n+1\right)}\right] \quad \text{(A13)}$$

$$C_7 = C_3 = -C_2 L^*_{\max} - C_6 L^*_{\max} + \frac{u^*_{up} {L^*_{\max}}^2}{2} - \frac{F^{1/n} L^*_{\max}\, 2n\left(Q_2/Q_1+1\right)\left(L^*_{\max}\right)\left(1-L^*_{\max}\right)^{1/n} - \left(1-L^*_{\max}\right)^{1/n} - \left(L^*_{\max}\right)^{n+1/n}}{2n+2} \quad \text{(A14)}$$

$$C_8 = \frac{F^{1/n}\left(n\right)\left(L^*_{\max}-y^*\right)^{3n+1/n}\left(Q_2/Q_1+1\right)}{\left(2n+1/n\right)\left(3n+1/n\right)\left(n+1\right)} \quad \text{(A15)}$$

$$C'_6 = \frac{B_1\left(F^{1/n}\left(n\right)\left(L^*_{\max}\right)^{n+1/n}\right)}{2n+2} \quad \text{(A16)}$$

$$C'_7 = \frac{\left(B_1 + B_1 n\right)}{n+1} \quad \text{(A17)}$$

$$C'_8 = C'_3 - \frac{A_1 {L^*_{\max}}^2\left[u^*_{up} + u^*_{up} n + F^{1/n} n\left(1-L^*_{\max}\right)^{1/n} - F^{1/n} n L^*_{\max}\left(\left(1-L^*_{\max}\right)^{1/n} + \left(L^*_{\max}\right)^{1/n}\right)\right]}{2n+2} \quad \text{(A18)}$$

$$C'_9 = -\frac{F^{1/n} n^2\left(F + Fn + A_1 n\right)}{\left(n+1\right)\left(3n+1\right)} \quad \text{(A19)}$$

where

$$B_1 = -\frac{F^{1/n}\left(n\right)\left(L^*_{\max}\right)^{2n+1/n}\left(F + Fn + A_1 n\right)}{\left(n+1\right)\left(2n+1\right)} \quad \text{(A20)}$$

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